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ON CLASS OF ESTIMATORS FOR POPULATION MEAN USING MULTI-AUXILIARY CHARACTERS IN THE PRESENCE OF NON-RESPONSE

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ABSTRACT

Two classes of estimators for the population mean of the study character using multi-auxiliary characters with known population means in presence of non-response have been proposed. The expressions for bias, mean square error and conditions for attaining minimum mean square error of the proposed classes of estimators have been obtained. An empirical study has also been given in support of the problem.

Key words: Population Mean; Bias; Mean square error; Non-response.

1. Introduction

The problem of estimating the population mean using multi-auxiliary characters with known population means has been considered by several authors [Olkin (1958), Shukla (1965,66), Rao and Mudholkar (1967), Mohanty (1967,70), Srivastava (1971), Khare and Srivastava (1980,81), Khare (1983), Srivastava and Jhaji (1983), Sahoo (1986), Sampath (1989)] when there is no non-response on sample units. However, it is a common experience that in sample surveys, the data may not be collected for all the units selected in the sample due to the problem of non-response. Frequently it has been observed in demographic surveys that some units selected in the sample may not respond on the study character (y) while the data on all the units are available for auxiliary characters (x). If there is non-response on some sample units then the estimators for population mean using auxiliary character in presence of non-response have been considered by Rao (1986, 90), Khare (2002) and Khare and Srivastava (1996, 97, 2000, 2002).

In the present context we have proposed two classes of estimators t_1 and t_1^* for the population mean of the study character using multi-auxiliary characters

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with known population means in the presence of non-response and studied their properties. The expressions for bias and mean square error of the proposed classes of estimators have been obtained. The conditions for minimising the mean square error of the proposed classes of estimators have also been obtained. The efficiency of the proposed classes of estimators with respect to the relevant estimator has been shown with the help of an empirical study.

2. The estimators

Let y and x_j (j = 1,2,...,p) be the main and auxiliary characters under study having non-negative ℓ-th value (Y_ℓ, $X_{j\ell}$; ℓ = 1,2,...,N and j = 1,2,...,p) with population mean $\overline{Y}$ and $\overline{X}_j$ (j = 1,2,...,p) respectively. The whole population is supposed to be divided in N_1 responding and N_2 non-responding units such that $N_1 + N_2 = N$. A sample of size n is drawn by using SRSWOR method of sampling from the population of N and it is observed that out of these n selected units, n_1 units respond and n_2 units do not respond. We have considered that the responding and non-responding units are same for study character as well as auxiliary characters. Further, from n_2 non-responding units, a subsample of size 'r' ($r = n_2/k$; $k > 1$) is drawn by making extra effort using SRSWOR technique of sub-sampling and consequently the estimator $\overline{y}^*$ [Hansen and Hurwitz (1946)] for the population mean $\overline{Y}$ is given by

$$\overline{y}^* = \frac{n_1}{n}\overline{y}_1 + \frac{n_2}{n}\overline{y}_2' \tag{2.1}$$

where $\overline{y}_1$ and $\overline{y}_2'$ are the sample means of characters y based on n_1 and r units respectively. The estimator $\overline{y}^*$ is unbiased and the variance of $\overline{y}^*$ is given by

$$V(\overline{y}^*) = \frac{1-f}{n} S_0^2 + \frac{W_2(k-1)}{n} S_{0(2)}^2 , \tag{2.2}$$

where $W_i = \frac{N_i}{N}$ (i = 1,2), $f = \frac{n}{N}$ and S_0^2, $S_{0(2)}^2$ denote the population mean square of y for the entire population and for the non-responding part of the population.

Similarly, the estimator $\overline{x}_j^*$ (j = 1,2,...,p) for the population means $\overline{X}_j$ (j = 1,2,...,p) is given by

$$\bar{x}_j^* = \frac{n_1}{n}\bar{x}_{j(1)} + \frac{n_2}{n}\bar{x}'_{j(2)}\,, \tag{2.3}$$

where $\bar{x}_{j(1)}$ and $\bar{x}'_{j(2)}$ are the sample means of character x_j based on n_1 and r units respectively. The variance of $\bar{x}_j^*$ is given by

$$V(\bar{x}_j^*) = \frac{1-f}{n}S_j^2 + \frac{W_2(k-1)}{n}S_{j(2)}^2\,, \tag{2.4}$$

where S_j^2 and $S_{j(2)}^2$ denote the population mean square of x_j for the entire population and for the non-responding part of the population respectively.

Now, we proposed the classes of estimators for population mean by utilizing the multi-auxiliary characters with known population means under two different cases which are given as follows:

Case I: Incomplete Information on y and x_j (j = 1,2,...,p)

When population means of x_1, x_2,..., x_p are known and only n_1 of the n units respond on both y and x_j (j = 1,2,...,p), then we propose a class of estimator t_1 for population mean $\bar{Y}$ using multi-auxiliary characters with known population means in presence of non-response which is as follows:

$$t_1 = g(v, \mathbf{u}')\,, \tag{2.5}$$

such that

$$g(\bar{Y}, \mathbf{e}') = \bar{Y}\,, \quad g_1(\bar{Y}, \mathbf{e}') = \left(\frac{\partial}{\partial v}g(v, \mathbf{u}')\right)_{(\bar{Y}, \mathbf{e}')} = 1\,, \tag{2.6}$$

where **u** and **e** denote the column vectors $(u_1, u_2, ..., u_p)'$ and $(1,1,...,1)'$ respectively. We also denote $v = \bar{y}^*$ and $u_j = \bar{x}_j^* / \bar{X}_j \quad (j = 1,2,\ldots,p)$.

Case II: Complete Information on x_j (j = 1, 2,...,p) But Incomplete Information on y

When population means of $x_1, x_2, ..., x_p$ are known and only n_1 of the n units responds on y but there is complete information on x_j ; j = 1,2,..,p for all the n units of the sample, then we propose a class of estimators t_1^* for population mean $\bar{Y}$ using multi-auxiliary characters with known population means in presence of non-response on y only which is given as follows:

$$t_1^* = h(v, \boldsymbol{\omega}')\,, \tag{2.7}$$

such that

$$h(\bar{Y},\mathbf{e}')=\bar{Y}\,,\quad h_1(\bar{Y},\mathbf{e}')=\left(\frac{\partial}{\partial v}h(v,\boldsymbol{\omega}')\right)_{(\bar{Y},\mathbf{e}')}=1\,, \qquad (2.8)$$

where $\boldsymbol{\omega}$ denotes the column vector $(\omega_1,\omega_2,...,\omega_p)$ and

$$\omega_j=\bar{x}_j/\bar{X}_j \quad (j = 1, 2,...,p)$$

The functions $g(v,\mathbf{u}')$ and $h(v,\boldsymbol{\omega}')$ also satisfy the following conditions -

(i) For any sampling design, whatever be the sample chosen, $(v,\mathbf{u}')$ and $(v,\boldsymbol{\omega}')$ assume values in a bounded closed convex subset D_1 and D_2 respectively of p+1 dimensional real space containing the point $(\bar{Y},\mathbf{e}')$.

(ii) The functions $g(v,\mathbf{u}')$ and $h(v,\boldsymbol{\omega}')$ and their first and second partial derivatives exist and are continuous and bounded in D_1 and D_2 respectively.

Here $[g_1(v,\mathbf{u}'),\ g_2(v,\mathbf{u}')]$ and $[h_1(v,\boldsymbol{\omega}'),\ h_2(v,\boldsymbol{\omega}')]$ denote the first partial derivatives of $g(v,\mathbf{u}')$ and $h(v,\boldsymbol{\omega}')$ with respect to [v, $\mathbf{u}'$] and [v, $\boldsymbol{\omega}'$] respectively. The second partial derivative of $g(v,\mathbf{u}')$, $h(v,\boldsymbol{\omega}')$ with respect to $\mathbf{u}'$ and $\boldsymbol{\omega}'$ are denoted by $g_{22}(v,\mathbf{u}')$, $h_{22}(v,\boldsymbol{\omega}')$ and first partial derivative of $g_2(v,\mathbf{u}')$ and $h_2(v,\boldsymbol{\omega}')$ with respect to v are denoted by $g_{12}(v,\mathbf{u}')$ and $h_{12}(v,\boldsymbol{\omega}')$.

On occurrence of the regularity conditions imposed on $g(v,\mathbf{u}')$ and $h(v,\boldsymbol{\omega}')$, it may be seen that the bias and mean square error of the estimators t_1 and t_1^* will always exist.

3. Bias and mean square error (MSE)

Expand the functions $g(v,\mathbf{u}')$ and $h(v,\boldsymbol{\omega}')$ about the point $(\bar{Y},\mathbf{e}')$ using Taylor's series up to second order partial derivatives, under the conditions (2.6) and (2.8), the expressions for bias and mean square error of the proposed classes of estimators t_1 and t_1^* for any sampling design up to the terms of order n^{-1} are given by

$$\text{Bias}\,(t_1)=E(v-\bar{Y})\,(\mathbf{u}-\mathbf{e})'\ g_{12}(v^*,\mathbf{u}^{*\prime})+\frac{1}{2}E(\mathbf{u}-\mathbf{e})'\,g_{22}(v^*,\mathbf{u}^{*\prime})\,(\mathbf{u}-\mathbf{e})\,, \qquad (3.1)$$

$$\text{MSE}\,(t_1)=V(\bar{y}^*)+2E(v-\bar{Y})\,(\mathbf{u}-\mathbf{e})'\ g_2(\bar{Y},\mathbf{e}')$$

$$+\ E(g_2(\overline{Y},\mathbf{e}'))'\ (\mathbf{u}-\mathbf{e})\ (\mathbf{u}-\mathbf{e})'\ g_2(\overline{Y},\mathbf{e}')\ , \tag{3.2}$$

$$\text{Bias}\ (t_1^*) = E(v-\overline{Y})\ (\omega-\mathbf{e})'\ h_{12}(v^*,\omega^{*\prime}) + \frac{1}{2} E(\omega-\mathbf{e})'\ h_{22}(v^*,\omega^{*\prime})\ (\omega-\mathbf{e}) \tag{3.3}$$

and

$$\text{MSE}\,(t_1^*) = V(\bar{y}^*) + 2E(v-\overline{Y})\ (\boldsymbol{\omega}-\mathbf{e})'\ h_2(\overline{Y},\mathbf{e}')$$

$$+\ E(h_2(\overline{Y},\mathbf{e}'))'\,(\boldsymbol{\omega}-\mathbf{e})\,(\boldsymbol{\omega}-\mathbf{e})'\ h_2(\overline{Y},\mathbf{e}')\ , \tag{3.4}$$

where $v^* = \overline{Y} + \phi\,(v-\overline{Y})$, $\mathbf{u}^* = \mathbf{e} + \boldsymbol{\phi}_1(\mathbf{u}-\mathbf{e})$, $\boldsymbol{\omega}^* = \mathbf{e} + \boldsymbol{\phi}_2(\boldsymbol{\omega}-\mathbf{e})$, while $\boldsymbol{\phi}_1$ and $\boldsymbol{\phi}_2$ denote the diagonal matrix of order $p \times p$ having j^{th} diagonal elements ϕ_{1j} and ϕ_{2j} ($j = 1,2,...,p$) respectively, such that

$$0 < \phi,\ \phi_{1j},\ \phi_{2j} < 1\ , \quad \forall\, j = 1,2,...,p.$$

The mean square error of t_1 and t_1^* will attain minimum value for

$$g_2(\overline{Y},\mathbf{e}') = -(E(\mathbf{u}-\mathbf{e})\,(\mathbf{u}-\mathbf{e})')^{-1}\,.\,E(v-\overline{Y})\,(\mathbf{u}-\mathbf{e}) \tag{3.5}$$

and

$$h_2(\overline{Y},\mathbf{e}') = -(E(\boldsymbol{\omega}-\mathbf{e})\,(\boldsymbol{\omega}-\mathbf{e})')^{-1}\,.\,E(v-\overline{Y})\,(\boldsymbol{\omega}-\mathbf{e}) \tag{3.6}$$

respectively. The minimum values of MSE(t) and $\text{MSE}\,(t_1^*)$ are given by

$$\text{MSE}(t_1)_{\min.} = V(\bar{y}^*) - E(v-\overline{Y})\,(\mathbf{u}-\mathbf{e})'\,(E(\mathbf{u}-\mathbf{e})\,(\mathbf{u}-\mathbf{e})')^{-1}\,E(v-\overline{Y})\,(\mathbf{u}-\mathbf{e}) \tag{3.7}$$

and

$$\text{MSE}(t_1^*)_{\min.} = V(\bar{y}^*) - E(v-\overline{Y})\,(\boldsymbol{\omega}-\mathbf{e})'\,(E(\boldsymbol{\omega}-\mathbf{e})\,(\boldsymbol{\omega}-\mathbf{e})')^{-1}\,E(v-Y)\,(\boldsymbol{\omega}-\mathbf{e}) \tag{3.8}$$

Considering SRSWOR method of sampling, let $\mathbf{A} = [a_{jj'}]$ and $\mathbf{A}_0 = [a_{0jj'}]$ are two $p \times p$ positive definite matrix such that

$$a_{jj'} = \frac{1-f}{n}\rho_{jj'}\,C_j\,C_{j'} + \frac{W_2(k-1)}{n}\rho_{jj'(2)}\,C'_j\,C'_{j'}$$

and

$$a_{0jj'} = \rho_{jj'}\,C_j\,C_{j'} \qquad \forall\ j \neq j' = 1,2,...,p$$

Also let $\mathbf{b} = (b_1, b_2 ..., b_p)'$ and $\mathbf{d} = (d_1, d_2 ..., d_p)'$ be two column vectors such that

$$b_j = \rho_j C_j \quad \text{and} \quad d_j = \rho_{j(2)} C'_j ,$$

where $C_j^2 = S_j^2 / \overline{X}_j^2$, $C_j'^2 = S_{j(2)}^2 / \overline{X}_j^2$.

Here, we define $\rho_{jj'}$ and $\rho_{jj'(2)}$ to be the correlation coefficients between x_j and $x_{j'}$ for the entire population and for the non-responding group of the population respectively while ρ_j and $\rho_{j(2)}$ to be the correlation coefficients between y and x_j for the entire and non-responding group of the population respectively.

Now, in the case of SRSWOR methods of sampling, the expressions for bias and mean square error of t_1 and t_1^* up to the terms of order (n^{-1}) are given as follows:

$$\text{Bias}(t_1) = \left(\frac{1-f}{n} S_0\, \mathbf{b} + \frac{W_2(k-1)}{n} S_{0(2)}\, \mathbf{d}\right)' g_{12}(v^*, \mathbf{u}^{*\prime})$$

$$+ \frac{1}{2} \text{trace}\, \mathbf{A}\ g_{22}(v^*, \mathbf{u}^{*\prime}) , \tag{3.9}$$

$$\text{MSE}(t_1) = V(\overline{y}^*) + (g_2(\overline{Y}, \mathbf{e}'))' \mathbf{A}\, g_2(\overline{Y}, \mathbf{e}')$$

$$+ 2\left(\frac{1-f}{n} S_0\, \mathbf{b} + \frac{W_2(k-1)}{n} S_{0(2)}\, \mathbf{d}\right)' g_2(\overline{Y}, \mathbf{e}') , \tag{3.10}$$

$$\text{Bias}(t_1^*) = \frac{1-f}{n}\left(S_0\, \mathbf{b}'\ h_{12}(v^*, \boldsymbol{\omega}^{*\prime}) + \frac{1}{2} \text{trace}\, \mathbf{A}_0\ h_{22}(v^*, \boldsymbol{\omega}^{*\prime})\right), \tag{3.11}$$

and

$$\text{MSE}(t_1^*) = V(\overline{y}^*) + \frac{1-f}{n}[(h_2(\overline{Y}, \mathbf{e}'))' \mathbf{A}_0\, h_2(\overline{Y}, \mathbf{e}') + 2S_0\, \mathbf{b}'\, h_2(\overline{Y}, \mathbf{e}')] \tag{3.12}$$

The values of $g_2(\overline{Y}, \mathbf{e}')$ and $h_2(\overline{Y}, \mathbf{e}')$ for which MSE(t_1) and MSE (t_1^*) will attain minimum value respectively are given by

$$g_2(\overline{Y}, \mathbf{e}') = -\mathbf{A}^{-1}\left(\frac{1-f}{n} S_0\, \mathbf{b} + \frac{W_2(k-1)}{n} S_{0(2)}\, \mathbf{d}\right) \tag{3.13}$$

and

$$h_2(\overline{Y}, \mathbf{e}') = -S_0 \mathbf{A}_0^{-1} \mathbf{b}. \tag{3.14}$$

The values of minimum mean square error for the estimators t_1 and t_1^* are given by

$$MSE(t_1)_{min.} = V(\overline{y}^*) - \left(\frac{1-f}{n} S_0 \mathbf{b} + \frac{W_2(k-1)}{n} S_{0(2)} \mathbf{d}\right)'$$

$$\mathbf{A}^{-1} \left(\frac{1-f}{n} S_0 \mathbf{b} + \frac{W_2(k-1)}{n} S_{0(2)} \mathbf{d}\right) \tag{3.15}$$

and

$$MSE(t_1^*)_{min.} = V(\overline{y}^*) - \left(\frac{1-f}{n}\right) S_0^2 \mathbf{b}' \mathbf{A}_0^{-1} \mathbf{b}. \tag{3.16}$$

4. Some particular members of the proposed class of estimators t_1 and t_1^*

The proposed classes of estimators t_1 and t_1^* are the wider class of estimators. All the members belonging to the classes of estimators t_1 and t_1^* attain minimum value of mean square error given in (3.15) and (3.16) if the conditions (3.13) and (3.14) are satisfied respectively. Now, we consider the members of the classes of estimators t_1 and t_1^* which are as follows:

$$T_{01} = v \exp\left[\sum_{j=1}^{p} \theta_{1j} \log u_j\right], \tag{4.1}$$

$$T_{02} = v \sum_{j=1}^{p} W_j u_j^{\theta_{2j}/W_j}, \qquad \sum_{i=1}^{p} W_j = 1. \tag{4.2}$$

$$T_{03} = \sum_{j=1}^{p} [W_j u_j^{\theta_{3j}/W_j}]\,[v + \beta_{1j}(u_j - 1)]. \tag{4.3}$$

$$T_{01}^{*} = v \exp\left[\sum_{j=1}^{p} \alpha_{1j} \log \omega_j\right], \tag{4.4}$$

$$T_{02}^{*} = v \sum_{j=1}^{p} W_j \, \omega_j^{\alpha_{2j}/W_j}, \qquad \sum_{i=1}^{p} W_j = 1. \tag{4.5}$$

and

$$T_{03}^{*} = \sum_{j=1}^{p} [W_j \, \omega_j^{\alpha_{3j}/W_j}] \, [v + \beta_{2j}(\omega_j - 1)] \tag{4.6}$$

The minimum mean square value of the classes of estimators t_1 and t_1^{*} can be obtained by putting the optimum value of the constants involved in the mean square error of the estimators which are the members of the proposed classes. Sometimes the members of the class attain minimum mean square error for some specified conditions which may or may not exist. In such cases these estimators are rarely used, when the specified conditions are not satisfied. Further, the proposed classes of estimators t_1 and t_1^{*} attain minimum value of mean square error if the conditions (3.13) and (3.14) are respectively satisfied. These conditions involve the unknown parameters, so for obtaining the required value of the constants, one may use the value of the parameters based on past data or experience. In case when past data is not available, one may estimate the required parameters based on preliminary sample. Reddy (1978) has shown that such values are stable overtime and region and do not affect the variance of the estimators up to the terms of order (n^{-1}) [Srivastava and Jhajj (1983)]. Therefore, the use of proposed classes of estimators is recommended in large scale sample surveys.

5. An empirical study

96 village wise population of rural area under Police-station – Singur, District – Hooghly, West Bengal has been taken under the study from the District Census Handbook 1981. The 25% villages (i.e. 24 villages) whose area is greater than 160 hectares have been considered as non-response group of the population. The number of agricultural labours in the village is taken as study character (y) while the area (in hectares) of the village, the number of cultivators in the village and the total population of the village are taken as auxiliary characters x_1, x_2 and x_3 respectively. The values of the parameters of the population under study are as follows:

$\overline{Y} = 137.9271$	$\overline{X}_1 = 144.8720$	$\overline{X}_2 = 185.2188$	$\overline{X}_3 = 1807.8958$
$S_0 = 182.5012$	$C_1 = 0.8115$	$C_2 = 1.0529$	$C_3 = 1.0591$
$S_{0(2)} = 287.4204$	$C'_1 = 0.9408$	$C'_2 = 1.4876$	$C'_3 = 1.5331$
$\rho_1 = 0.773$	$\rho_{1(2)} = 0.724$	$\rho_{12} = 0.819$	$\rho_{12(2)} = 0.724$
$\rho_2 = 0.786$	$\rho_{2(2)} = 0.787$	$\rho_{13} = 0.877$	$\rho_{13(2)} = 0.853$
$\rho_3 = 0.904$	$\rho_{3(2)} = 0.895$	$\rho_{23} = 0.906$	$\rho_{23(2)} = 0.891$

To study the performance of the proposed classes of estimators t_1 and t_1^* with respect to $\overline{y}^*$, we consider the estimators

$$T_{01} = v \exp\left[\sum_{j=1}^{p} \theta_{1j} \log u_j\right],$$

and

$$T_{01}^* = v \exp\left[\sum_{j=1}^{p} \alpha_{1j} \log \omega_j\right],$$

which are the members of the proposed classes of estimators t_1 and t_1^* respectively. The optimum values of the constant θ_{1j} and α_{1j} involved in T_{01} and T_{01}^* can be respectively calculated by the equation (3.13) and (3.14), which are as follows:

$$T_{01}\begin{cases}\text{For } k=4 \begin{cases} T_{01}\ (p=1):\ \theta_{11}=-1.4834\\ T_{01}\ (p=2):\ \theta_{11}=-0.6614,\ \theta_{12}=-0.7372\\ T_{01}\ (p=3):\ \theta_{11}=0.2897,\ \theta_{12}=0.1208,\ \theta_{13}=-1.4602\end{cases}\\[2ex] \text{For } k=3 \begin{cases} T_{01}\ (p=1):\ \theta_{11}=-1.4487\\ T_{01}\ (p=2):\ \theta_{11}=-0.6481,\ \theta_{12}=-0.7234\\ T_{01}\ (p=3):\ \theta_{11}=0.2684,\ \theta_{12}=0.1253,\ \theta_{13}=-1.4481\end{cases}\\[2ex] \text{For } k=2 \begin{cases} T_{01}\ (p=1):\ \theta_{11}=-1.3889\\ T_{01}\ (p=2):\ \theta_{11}=-0.6316,\ \theta_{12}=-0.6931\\ T_{01}\ (p=3):\ \theta_{11}=0.2271,\ \theta_{12}=0.1363,\ \theta_{13}=-1.4252\end{cases}\end{cases}$$

and

$$T_{01}^{*} \begin{cases} T_{01}^{*}\ (p=1) : \alpha_{11} = -1.2606 \\ T_{01}^{*}\ (p=2) : \alpha_{11} = -0.6447,\ \alpha_{12} = -0.5796 \\ T_{01}^{*}\ (p=3) : \alpha_{11} = 0.1013,\ \alpha_{12} = 0.1838,\ \alpha_{13} = -1.3589 \end{cases}$$

The mean square error and relative efficiency of t_1 and t_1^* with respect to $\bar{y}^*$ for one, two and three auxiliary characters with different values of the subsampling fraction (1/k) are given in Table 1.

Table 1. R.E.(·) in % with respect to $\bar{y}^*$ for different values of k

Estimators	Auxiliary character(s)	N = 96, n = 40					
		1/k					
		1/4		1/3		1/2	
$\bar{y}^*$	–	100.00	(2034.5580)*	100.00	(1518.2425)	100.00	(1001.9270)
T_{01}	x_1	215.97	(942.0470)	218.23	(695.7218)	223.76	(447.7739)
	x_1,x_2	298.33	(681.9845)	297.17	(510.8925)	296.09	(338.3828)
	x_1,x_2,x_3	525.21	(387.3806)	527.35	(287.8993)	532.49	(188.1597)
T_{01}^*	x_1	116.64	(1744.3606)	123.63	(1228.0451)	140.77	(711.7296)
	x_1,x_2	118.97	(1710.0916)	127.18	(1193.7761)	147.90	(667.4606)
	x_1,x_2,x_3	124.35	(1636.1917)	135.57	(1119.8762)	166.06	(603.3607)

**Figures in parenthesis represent MSE(·).*

From the Table 1, it is very clear that the mean square error of T_{01} and T_{01}^* decreases as the numbers of auxiliary characters and subsampling fractions increase. The estimators T_{01} and T_{01}^* are more efficient than $\bar{y}^*$ for different values of k while comparing T_{01} with T_{01}^* it is observed that the estimator T_{01} is more efficient than T_{01}^* and their efficiencies are increasing with respect to $\bar{y}^*$ when numbers of auxiliary characters and subsampling fractions increase. Hence we conclude that the efficiency of all the members of the class of estimators t_1 or t_1^* with respect to $\bar{y}^*$ can be increased by increasing the numbers of auxiliary characters as well as by increasing the value of subsampling fractions to be selected from non-responding group.

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